Maths Holiday Homework

Assignment Number-1

Matrix

1) If $\begin{bmatrix} x+2y & -y \\ 3x & 4 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 6 & 4 \end{bmatrix}$, find x and y

2) If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then for what value of α, A is an identity matrix?

3) Find the value of $x+y$ from the following equation
$$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$

4) If A is a square matrix such that $A^2 = A$, then write the value of $(I+A)^2 - 3A$.

5) If the matrix $A = \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}$ and $A^2 = \lambda A$, then write the value of λ

6) If A is a square matrix such that $A^2 = A$, then write the value of $7A - (I+A)^3$, where I is the identity matrix

7) Write the value of $x-y+z$ from the following equation
$$\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$

8) Solve for x: $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$

9) $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, show that $f(A) = 0$ where $f(x) = x^2 - 2x - 3$

10) $A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$, $0 < x < \frac{\pi}{2}$ and $A + A' = I$, then find the value of x

11) If $A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$, prove that $A^n = \begin{bmatrix} \cos nx & \sin nx \\ -\sin nx & \cos nx \end{bmatrix}$ for all positive integer n.

12) If $A = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$, prove by induction that
$(aI + bA)^n = a^n I + n a^{n-1} b A$, where I is the identity matrix of order 2 and n is positive integer

13) If A and B are symmetric matrix of same order, Prove that AB is symmetric if and only if A and B commute.

14) If A is 3×3 invertible matrix, then show that for any scalar k (non zero) kA is invertible and
$(kA)^{-1} = \frac{1}{k} A^{-1}$

15) If A and B are two matrices of the order $3\times m$ and $3\times n$, respectively and $m = n$, then write the order of matrix $5A - 2B$

16) If A is a matrix of order $m\times n$ and B is a matrix such that AB' and $B'A$ both are defined then write the order of matrix B

17) Construct a 3×2 matrix A whose elements are given by $a_{ij} = e^{ix} \sin jx$

18) For what value of x and y matrices $\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix}$ and $\begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$ are equal.

19) Find x : if $x\begin{bmatrix} 2x & 2 \\ 3 & x \end{bmatrix} + 2\begin{bmatrix} 8 & 5x \\ 4 & 4x \end{bmatrix} = 2\begin{bmatrix} x^2+8 & 24 \\ 10 & 6x \end{bmatrix}$

20) Find the matrix A if

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

21) Find A, if $\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} A = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$

22) If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ and I is the identity matrix of order 2 then find λ, μ so that $A^2 = \lambda A + \mu I$

23) Prove by the principle of mathematical induction for any square matrix A, $(A')^n = (A^n)'$

24) If $A = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $x^2 = -1$

then show that $(A+B)^2 = A^2 + B^2$

25) $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$. Then show that $A^2 - 4A + 7I = 0$

using this result calculate A^5.

26) Represent matrix $A = \begin{bmatrix} 5 & 3 & 1 \\ 2 & 8 & 9 \\ 1 & 4 & 3 \end{bmatrix}$ as the sum of symmetric and skew symmetric matrix

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Assignment No-2
Determinants

27) Find the value of x, if matrix $\begin{bmatrix} 2x+5 & 3 \\ 5x+2 & 9 \end{bmatrix}$ is singular.

28) Find x if $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$

29) If A is a matrix of order 3×3 and $|A|=4$ find
(i) $|3A|$ (ii) $|\text{adj}\,A|$ (iii) $|A(\text{adj}\,A)|$

30) Without expanding Evaluate the following

$$\begin{vmatrix} (2^x+2^{-x})^2 & (2^x-2^{-x})^2 & 1 \\ (3^x+3^{-x})^2 & (3^x-3^{-x})^2 & 1 \\ (4^x+4^{-x})^2 & (4^x-4^{-x})^2 & 1 \end{vmatrix}$$

31) Evaluate $\begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 75^\circ & \cos 75^\circ \end{vmatrix}$

32) If $x \in R$ $0 \le x \le \frac{\pi}{2}$ and $\begin{vmatrix} 2\sin x & -1 \\ 1 & \sin x \end{vmatrix} = \begin{vmatrix} 3 & 0 \\ -4 & \sin x \end{vmatrix}$ find x

33) There are two values of x which make determinant

$$\Delta = \begin{vmatrix} 1 & -2 & 5 \\ 2 & x & -1 \\ 0 & 4 & 2x \end{vmatrix} = 86$$ find the sum of these numbers

34) If x is a real number, then show that $\begin{vmatrix} 1 & \sin x & 1 \\ -\sin x & 1 & \sin x \\ -1 & \sin x & 1 \end{vmatrix}$ lies between 2 and 4

35) Without expanding show that

(i) $\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix} = 0$ (ii) $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix} = 0$

Solve the following using properties of determinant

36) $\begin{vmatrix} x+2 & x+6 & x-1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix} = 0$

37) $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$

38) $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$

Using properties of determinant prove the following (Q-39 to 47)

39) $\begin{vmatrix} 1 & a & b \\ -a & 1 & c \\ -b & -c & 1 \end{vmatrix} = 1+a^2+b^2+c^2$

40) $\begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & x(x+1) \\ 3(1-x) & x(x-1)(x-2) & x(x+1)(x-1) \end{vmatrix} = 6x^2(1-x^2)$

41) $\begin{vmatrix} (a+1)(a+2) & (a+2) & 1 \\ (a+2)(a+3) & (a+3) & 1 \\ (a+3)(a+4) & (a+4) & 1 \end{vmatrix} = 2$

42) $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$

43) $\begin{vmatrix} 2ab & a^2 & b^2 \\ a^2 & b^2 & 2ab \\ b^2 & 2ab & a^2 \end{vmatrix} = -(a^3+b^3)^2$

44) $\begin{vmatrix} x & p & q \\ p & x & q \\ q & q & x \end{vmatrix} = (x-p)(x^2+px-2q^2)$

45) $\begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (c+a)^2 & bc \\ ca & bc & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$

46) $\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^3$

47) $\begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 2x \\ \sin^2 x & 1+\cos^2 x & 4\sin 2x \\ \sin^2 x & \cos^2 x & 1+4\sin 2x \end{vmatrix} = 2+4\sin 2x$

48) If x, y, z are real numbers such that $x+y+z=\pi$, then find the value of $\begin{vmatrix} \sin(x+y+z) & \sin(x+z) & \cos z \\ -\sin y & 0 & \tan x \\ \cos(x+y) & \tan(y+z) & 0 \end{vmatrix}$

49) If A and B are square matrix of same order such that $|A| = -2$ and $AB = I$ find $|B|$

50) If A and B are square matrix of order 3 such that $|A| = -1$ and $|B| = 3$ find $|3AB|$ and $|A \cdot A'|$

51) If $a \neq b \neq c$ and $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$ then using properties of determinant prove that $a+b+c=0$

52) In a triangle ABC if $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & 1+\sin C \\ \sin A+\sin^2 A & \sin B+\sin^2 B & \sin C+\sin^2 C \end{vmatrix} = 0$ then show that ΔABC is an isosceles triangle

53) If p, q, r are not in G.P. and

$$\begin{vmatrix} 1 & q/p & \alpha + q/p \\ 1 & r/p & \alpha + r/p \\ p\alpha+q & q\alpha+r & 0 \end{vmatrix} = 0$$ show that

$p\alpha^2 + 2q\alpha + r = 0$

54) Using properties of determinants, find the value of
$$\begin{vmatrix} 1 & 1 & 1 \\ {}^{n}C_1 & {}^{n+2}C_1 & {}^{n+4}C_1 \\ {}^{n}C_2 & {}^{n+2}C_2 & {}^{n+4}C_2 \end{vmatrix}$$

55) A is a square matrix of order 3 and $|A| = 5$ find $|2A|$, $|A(\text{adj} A)|$, $|\text{adj} A|$ and $|A^{-1}|$

56) Show that the points $(a+5, a-4)$, $(a-2, a+3)$ and (a, a) do not lie in straight line

57) In a triangle ABC, if
$$\begin{vmatrix} 1 & 1 & 1 \\ 1+\cos A & 1+\cos B & 1+\cos C \\ \cos A+\cos^2 A & \cos B+\cos^2 B & \cos C+\cos^2 C \end{vmatrix} = 0$$
then prove that ABC is an isosceles triangle

Assignment - 3

Inverse trigonometric functions

58) Evaluate $\sin\left[\cot^{-1}\cot\left(\frac{17\pi}{3}\right)\right]$

59) Evaluate $\sin\left[2\cos^{-1}\left(-\frac{3}{5}\right)\right]$

60) Write the range of one branch of $\sin^{-1}x$, other than principle branch

61) Evaluate $\cos\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)+\frac{\pi}{6}\right]$

62) Evaluate $\tan^{-1}\left(\tan\frac{5\pi}{6}\right) + \cos^{-1}\left(\cos\frac{13\pi}{6}\right)$

63) Evaluate $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$

64) Prove that $2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right) = \cos^{-1}\left(\frac{a\cos x+b}{a+b\cos x}\right)$

65) Prove that $\cos^{-1}\left(\frac{\cos\alpha+\cos\beta}{1+\cos\alpha\cdot\cos\beta}\right) = 2\tan^{-1}\left(\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\right)$

66) Prove that: $\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]=\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2$

67) Prove that: $\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right)=\frac{2b}{a}$

68) Simplify $\cos^{-1}\left(\frac{3}{5}\cos x+\frac{4}{5}\sin x\right)$

69) If $y=\cot^{-1}(\sqrt{\cos x})-\tan^{-1}(\sqrt{\cos x})$, prove that $\sin y=\tan^2\frac{x}{2}$

70) Prove that

$$\frac{1}{2}\tan^{-1}x=\cos^{-1}\left[\sqrt{\frac{1+\sqrt{1+x^2}}{2\sqrt{1+x^2}}}\right]$$

71) Prove that $\sec^2(\tan^{-1}2)+\operatorname{cosec}^2(\cot^{-1}3)=15$

72) Simplify $\cos\left[\tan^{-1}\left\{\sin(\cot^{-1}x)\right\}\right]$

73) Solve $3\sin^{-1}\left(\frac{2x}{1+x^2}\right)-4\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)+2\tan^{-1}\frac{2x}{1-x^2}=\frac{\pi}{3}$

74) $2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}=\sin^{-1}\frac{31}{25\sqrt{2}}$

75) Prove that $\tan\left(2\tan^{-1}\frac{1}{5}+\frac{\pi}{4}\right)=\frac{17}{7}$

76) Solve $\tan^{-1}\left(\frac{x-5}{x-6}\right)+\tan^{-1}\left(\frac{x+5}{x+6}\right)=\frac{\pi}{4}$

77) Prove that $\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}=\sin^{-1}\frac{1}{\sqrt{5}}$

78) $4\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{239}=\frac{\pi}{4}$

79) Prove that $\sin^{-1}\frac{x}{\sqrt{1+x^2}}+\cos^{-1}\frac{x+1}{\sqrt{x^2+2x+2}}=\tan^{-1}(x^2+x+1)$

80) Find the value of $\cos\left[2\cos^{-1}x+\sin^{-1}x\right]$ at $x=\frac{1}{5}$

Assignment No - 4

Derivatives

Differentiate the following with respect to x

81) $\sin\left(\sqrt{\sin x + \cos x}\right)$

82) $\left(\dfrac{2\tan x}{\tan x + \cos x}\right)^2$

83) $\sqrt{\tan\sqrt{x}}$

84) $\sqrt{\dfrac{\sec x + 1}{\sec x - 1}}$

85) $\sqrt{\dfrac{1 - \tan x}{1 + \tan x}}$

86) $\dfrac{\sqrt{x^2+1} - x}{\sqrt{x^2+1} + x}$

87) $|x^2 - 5|$

88) $\sin\sqrt{x} + \cos^2\sqrt{x}$

89) $\dfrac{\sqrt{a+x} - \sqrt{a-x}}{\sqrt{a+x} + \sqrt{a-x}}$

90) $\dfrac{\sqrt{x^2+1} + \sqrt{x^2-1}}{\sqrt{x^2+1} - \sqrt{x^2-1}}$

91) $\sin^m x \cos^n x$

92) $\sin^n(ax^2 + bx + c)$

93) $\tan^{-1}(\sec x + \tan x)$

94) $\sin^{-1}\left(\dfrac{3\sin x + 4\cos x}{5}\right)$

95) $\sin^{-1}\left(\dfrac{\sqrt{1+x} - \sqrt{1-x}}{2}\right)$

96) $\sin^{-1}\left(\dfrac{5x + 12\sqrt{1-x^2}}{13}\right)$

97) $\sin\left[2\tan^{-1}\sqrt{\dfrac{1-x}{1+x}}\right]$

98) $\tan^{-1}\left[\dfrac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right]$

99) $\tan^{-1}\left[\dfrac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}}\right]$

100) $\sin^{-1}\left[x\sqrt{1-x} - \sqrt{x}\left(\sqrt{1-x^2}\right)\right]$

101) $\tan^{-1}\left(\dfrac{ax - b}{a + bx}\right)$

102) $\tan^{-1}\left(\dfrac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$

103) $\cos^{-1}\left(\dfrac{1-x}{1+x}\right)$

104) $\sin^{-1}\left(\dfrac{1}{\sqrt{x+1}}\right)$

105) $x(\sin^{-1}x)^2 + 2\sqrt{1-x^2}\,\sin^{-1}x - 2x$

106) $y = \left(x+\sqrt{x^2+a^2}\right)^n$, prove that $\dfrac{dy}{dx} = \dfrac{ny}{\sqrt{x^2+a^2}}$

107) $y = \left(x+\sqrt{x^2-1}\right)^m$, prove that $(x^2-1)\left(\dfrac{dy}{dx}\right)^2 = m^2y^2$

108) If $f(x) = \sqrt{x^2+1}$, $g(x) = \dfrac{x+1}{x^2+1}$ and $h(x) = 2x-3$

then find $f'[h'\{g'(x)\}]$

109) Prove that $\dfrac{d}{dx}\left\{\dfrac{x}{2}\sqrt{a^2-x^2} + \dfrac{a^2}{2}\sin^{-1}\dfrac{x}{a}\right] = \sqrt{a^2-x^2}$

110) Prove that $\dfrac{d}{dx}\left\{\dfrac{x}{2}\sqrt{x^2+a^2} + \dfrac{a^2}{2}\log\left(x+\sqrt{x^2+a^2}\right)\right\} = \sqrt{x^2+a^2}$

111) Prove that $\dfrac{d}{dx}\left\{\dfrac{x}{2}\sqrt{x^2-a^2} - \dfrac{a^2}{2}\log\left(x+\sqrt{x^2-a^2}\right)\right] = \sqrt{x^2-a^2}$

112) $y = \tan^{-1}\dfrac{5ax}{a^2-6x^2}$, prove that $\dfrac{dy}{dx} = \dfrac{3a}{a^2+9x^2} + \dfrac{2a}{a^2+4x^2}$

113) $y = \dfrac{\sin^{-1}x}{\sqrt{1-x^2}}$ prove that $(1-x^2)\dfrac{dy}{dx} - xy = 1$

114) If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, $x \neq y$ then prove that

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$

115) If $y = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + \cdots \infty}}}$ prove that

$$(1-2y)\frac{dy}{dx} = \sin x$$

116) If $y = \sqrt{x+\sqrt{x+\sqrt{x+\cdots\infty}}}$ prove that

$$(1-2y)\frac{dy}{dx} = \sin x$$

117) If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \cdots \infty}}}$, prove that

$$(2y-1)\frac{dy}{dx} = \cos x$$

118) $y = \sqrt{\tan x + \sqrt{\tan x + \sqrt{\tan x + \cdots \infty}}}$

prove that $(2y-1)\dfrac{dy}{dx} = \sec^2 x$

119) Differentiate $\sin^{-1}\left(\dfrac{2^{x+1}}{1+4^x}\right)$ with respect to x

120) Differentiate $\sin^{-1}\left(\dfrac{2^{x+1}.3^x}{1+36^x}\right)$ with respect to x

121) If $\log\sqrt{x^2+y^2} = \tan^{-1}\left(\dfrac{x}{y}\right)$ show that $\dfrac{dy}{dx} = \dfrac{y-x}{y+x}$

122) If $y = \sqrt{x^2+1} = \log\left(\sqrt{x^2+1} - x\right)$, prove that

$(x^2+1)\dfrac{dy}{dx} + xy + 1 = 0$

123) If $y = \log\left(\dfrac{x+\sqrt{x^2+1}}{\sqrt{x^2+1}}\right)$, prove that $(x^2+1)\dfrac{dy}{dx} + xy = 1$

124) if $x = e^{x/y}$ prove that $\dfrac{dy}{dx} = \dfrac{x-y}{x\log x}$

125) If $x^y = e^{x-y}$, prove that $\dfrac{dy}{dx} = \dfrac{\log x}{(\log xe)^2}$

126) Differentiate $\tan^{-1}\left(\dfrac{x}{1+\sqrt{1-x^2}}\right)$ w.r.t. $\sec^{-1}\left(\dfrac{1}{2x^2-1}\right)$.

127) Find $\dfrac{dy}{dx}$ if $x = \dfrac{1+\log t}{t^2}$ $y = \dfrac{3+2\log t}{t}$

128) $y = \tan^{-1}x$ find $\dfrac{d^2y}{dx^2}$ in terms of y alone

129) If $y = \left(\log\left(x+\sqrt{x^2+a^2}\right)\right)^2$, prove that

$$(x^2+a^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 2$$

130) $y = (a+bx)e^{cx}$, prove that $y_2 - 2cy_1 + c^2y = 0$

131) If $x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t)$, $0 < t < \frac{\pi}{2}$,

find $\frac{d^2x}{dt^2}$, $\frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$

132) $x = \sin t$, $y = \sin pt$, prove that $(1-x^2)y'' - xy' + p^2y = 0$

133) If $x^m y^n = (x+y)^{m+n}$, then prove that

(i) $\frac{dy}{dx} = \frac{y}{x}$ (ii) $\frac{d^2y}{dx^2} = 0$

134) If $y = \tan x + \sec x$, prove that $\frac{d^2y}{dx^2} = \frac{\cos x}{(1-\sin x)^2}$.

135) If $x = 3\sin t - \sin 3t$, $y = 3\cos t - \cos 3t$, find $\frac{dy}{dx}$ at $t = \frac{\pi}{3}$

136) Differentiate $x^{\sin^{-1}x}$ with respect to $\sin^{-1}x$

137) $y = x^{x^{x^{\cdots\infty}}}$, prove that $\frac{dy}{dx} = \frac{y^2}{x(1-y\log x)}$

138) find $\frac{dy}{dx}$, if $y^x + x^y + x^x = a^b$

139) If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$

140) $y = e^{x+e^{x+e^{x\cdots\infty}}}$, prove that

$\frac{dy}{dx} = \frac{y}{1-y}$.